

Continuous area spectrum in regular black hole

Hidefumi Nomura^{1*} and Takashi Tamaki^{2†}

¹*Department of Physics, Waseda University, 3-4-1 Okubo, Shinjuku, Tokyo 169-8555, Japan*

²*Department of Physics, Kyoto University, 606-8501, Japan*

(Dated: February 1, 2008)

We investigate highly damped quasinormal modes of regular black hole coupled to nonlinear electrodynamics. Using the WKB approximation combined with complex-integration technique, we show that the real part of the frequency disappears in the highly damped limit. If we use the Bohr's correspondence principle, the area spectrum of this black hole is continuous. We discuss its implication in the loop quantum gravity.

PACS numbers: 04.70.Bw, 04.30.Db, 04.70.Dy

I. INTRODUCTION

Quasinormal modes (QNMs) of black holes are solutions of the classical perturbation equations in the gravitational background with the specific boundary conditions for purely outgoing waves at infinity and purely ingoing at the event horizon. QNMs are intrinsic quantities that characterize black holes. These are expected to dominate the emitted radiation in many dynamical processes involving a black hole at late times. Since QNMs are also expected to reveal the information about parameters of black holes, it is important in detecting them from the astrophysical viewpoint. For a review, see, e.g., [1].

During the last few years, they have also attracted much attention in the context of quantum gravity. This is related to the area quantization of black holes discussed by Bekenstein [2]. First, we identify the real part of the highly damped QNMs as a minimum change of the black hole mass based on the Bohr's correspondence principle [3]. For Schwarzschild black hole, we have [4]

$$\text{Re}(\omega) = T_H \ln 3 \quad \text{as} \quad |\text{Im}(\omega)| \rightarrow \infty. \quad (1.1)$$

Applying the first law of black hole thermodynamics, we obtain

$$dA = 4dM/T_H = 4\ln 3 \quad (1.2)$$

where $dM = dE = \text{Re}(\omega)$. The reason why it has been paid attention is the relation to the loop quantum gravity where the area spectrum is given by [5]

$$A = 8\pi\gamma \sum_i \sqrt{j_i(j_i + 1)}, \quad (1.3)$$

where γ is the Immirzi parameter related to an ambiguity in the choice of canonically conjugate variables [6]. The sum is added up all intersections between a surface and a spin network carrying a label $j = 0, 1/2, 1, 3/2, \dots$ reflecting the $\text{SU}(2)$ nature of the gauge group. The statistical origin of the black hole entropy S is also derived

in [7]. The idea is to identify the minimum element in (1.3) A_{\min} and (1.2), i.e.,

$$dA = 4 \ln 3 = 8\pi\gamma \sqrt{j_{\min}(j_{\min} + 1)}, \quad (1.4)$$

and use the relation $S = A/4$ [8]. Then, j_{\min} is determined as 1, which is consistent with the requirement that j is half-integer. Since this consistency seems meaningful, various arguments has been done [9, 10, 11, 12, 13, 14].

The main reasons opposing this idea are summarized as follows. (i) Other black holes, such as Reissner-Nordström black hole [4, 15, 16], Schwarzschild de-sitter (dS) black hole [17, 18], Kerr black hole [19], and also in d-dimensional Schwarzschild and Reissner-Nordström with a cosmological constant [20] do not have above consistency. (ii) Original calculation of the black hole entropy has a mistake [21, 22]. The corrected entropy suggests that j_{\min} determined above way is not half-integer.

On the other hand, there are also reasons supporting this idea. (i) Schwarzschild black hole in other dimensions has the relation (1.2) [4, 23, 24, 25]. Surprisingly, single-horizon black holes, such as dilatonic black hole [26] shares the relation (1.2) [27] as it has been suggested in [28, 29]. This has been confirmed in other way in [30, 31]. This universality suggests meaningful. (ii) The black hole entropy has been reexamined based on the idea that spherical symmetry should be reflected in the number counting of microstates for spherically symmetric black holes [32]. In this case, original consistency that $j_{\min} = 1$ has been recovered.

As shown above, this is still controversial. Therefore, we need further discussion and study from both QNMs and the loop quantum gravity. It is also interesting that QNMs of AdS black holes have a direct interpretation in terms of the dual conformal field theory (CFT) [33, 34] according to the AdS/CFT correspondence [35, 36]. There is also a possibility that QNMs of AdS black holes play an important role in determining the microstates of black holes [37]. Its application to the general case is still hypothetical [38]. However, it is stimulating that two candidates of the quantum gravity suggest the important role in QNMs independently.

Therefore, it is natural in examining QNMs of black holes with quantum gravity motivated model, for example, Gauss-Bonnet black hole [39]. In such a model, it is

*Electronic address: nomura@gravity.phys.waseda.ac.jp

†Electronic address: tamaki@tap.scphys.kyoto-u.ac.jp

expected to be singularity-free. In this paper, we focus on the highly damped QNMs of “regular” (no singularity inside the horizon) black hole coupled to the nonlinear electrodynamics satisfying the weak energy condition [40, 41]. We also discuss its interpretation from the loop quantum gravity.

II. HIGHLY DAMPED QNMS OF REGULAR BLACK HOLE

Here, we investigate the asymptotic QNMs of regular black hole using the WKB analysis combined with complex-integration technique following [15]. We use the line element of the regular black hole obtained in Einstein gravity coupled with nonlinear electrodynamics proposed in [40] which can be expressed as

$$ds^2 = - \left(1 - \frac{2Mr^2}{(r^2 + q^2)^{3/2}} + \frac{q^2 r^2}{(r^2 + q^2)^2} \right) dt^2 + \left(1 - \frac{2Mr^2}{(r^2 + q^2)^{3/2}} + \frac{q^2 r^2}{(r^2 + q^2)^2} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad (2.1)$$

where the associated electric field E is given by

$$E = q r^4 \left(\frac{r^2 - 5q^2}{(r^2 + q^2)^4} + \frac{15}{2} \frac{M}{(r^2 + q^2)^{7/2}} \right). \quad (2.2)$$

Note that this solution asymptotically behaves as the Reissner–Nordström solution,

$$\begin{aligned} -g_{tt} &= 1 - 2M/r + q^2/r^2 + O(1/r^3), \\ E &= q/r^2 + O(1/r^3). \end{aligned} \quad (2.3)$$

Thus, the parameters M and q are related correspondingly with the mass and the electric charge. If $|q| < 2s_c M$ ($s_c \simeq 0.317$ see [40]), this expresses a regular charged black hole which has inner horizon r_- and event horizon r_+ . We concentrate on this solution from now on. We define

$$g(r) \equiv 1 - \frac{2m(r)}{r}, \quad (2.4)$$

where

$$m(r) = \frac{Mr^3}{(r^2 + q^2)^{3/2}} - \frac{q^2 r^3}{2(r^2 + q^2)^2}. \quad (2.5)$$

Notice the relation to the Hawking temperature T_H is [28]

$$g'(r_+) = 4\pi T_H, \quad (2.6)$$

where $' := d/dr$. The perturbation (Regge–Wheeler) equation for (2.1), with the time dependence $\exp(-i\omega t)$, is

$$\frac{d^2 \psi}{dr_*^2} + [\omega^2 - V(r)]\psi = 0, \quad (2.7)$$

where r_* denotes the tortoise coordinate given by

$$\frac{dr_*}{dr} = \frac{1}{g(r)}. \quad (2.8)$$

and the Regge–Wheeler potential is given by [28]

$$\begin{aligned} V(r) &= g \left(\frac{l(l+1)}{r^2} + (1-k^2) \frac{2m}{r^3} \right. \\ &\quad \left. + (1-k) \left(\frac{g'}{r} - \frac{2m}{r^3} \right) \right) \end{aligned} \quad (2.9)$$

$k = 0, 1$, and 2 for scalar, electromagnetic, and odd parity gravitational perturbations, respectively. We impose the boundary conditions, which are purely outgoing plane waves at spatial infinity and purely ingoing plane waves at the horizons, on ψ later. Introducing $\Psi = g^{1/2}(r)\psi$, we can rewrite (2.7) as

$$\frac{d^2 \Psi}{dr^2} + R(r)\Psi = 0, \quad (2.10)$$

where

$$R(r) = g^{-2} \left(\omega^2 - V(r) + \frac{g'^2}{4} - \frac{gg''}{2} \right). \quad (2.11)$$

From now, we consider the WKB analysis combined with complex-integration technique, which is a good approximation in the limit $\text{Im}(\omega) \rightarrow -\infty$. We seek for the WKB condition which corresponds to the monodromy condition of Motl and Neitzke [4]. The two WKB solutions to an equation of form (2.10) can be written as

$$\Psi_{1,2}^{(s)}(r) = Q(r)^{-1/2} \exp \left(\pm i \int_s^r Q(r') dr' \right), \quad (2.12)$$

with $Q^2 \equiv R + (\text{extra term})$. The zeros and poles of the function Q^2 play a central role in our complex analysis [15]. Notice that the zeros approach $r^2 \simeq -q^2$ in the limit $\text{Im}(\omega) \rightarrow -\infty$. Here, We choose the (extra term) for Ψ to behave properly near $r^2 \simeq -q^2$.

Expanding near $r^2 \simeq -q^2$, we obtain

$$R(r) \simeq \frac{(r^2 + q^2)^4}{q^8} (\omega^2 - 8 \frac{q^4 r^6}{(r^2 + q^2)^6}). \quad (2.13)$$

It is independent of k . Therefore, the perturbation equation near $r^2 \simeq -q^2$ becomes

$$\frac{d^2 \Psi}{dr^2} + \frac{8q^2}{(r^2 + q^2)^2} \Psi = 0. \quad (2.14)$$

Thus, the asymptotic solution can be written as

$$\Psi \simeq (r \mp iq)^{1/2 \pm 3/2} \quad (r \simeq \pm iq). \quad (2.15)$$

Then we should choose

$$\begin{aligned} Q^2 &= R + \frac{q^2}{(r^2 + q^2)^2} \\ &\simeq \frac{(r^2 + q^2)^4}{q^4 r^4} (\omega^2 - 9 \frac{q^4 r^6}{(r^2 + q^2)^6}) \end{aligned} \quad (2.16)$$

for the WKB solution (2.12) to coincide with (2.15) near $r \simeq \pm iq$. This is analogous to the “Langer modification” $l(l+1) \rightarrow (l+1/2)^2$ that is used in the WKB analysis of radial quantum problems [42].

Then, we can find that the function Q^2 has four second order poles ($r = r_-, r_+, \pm iq$) and twelve zeros (around $r = \pm iq$). We depict these poles and zeros in Fig. 1. We explain the technique that is crucial for our analysis. From each simple zero of Q^2 emanates three so-called “Stokes lines”. Along each of these contours, $Q(r)dr$ is purely imaginary, which means that one of the two solutions grows exponentially while the second solution decays, as we move this line. In other words, one of the solutions is exponentially dominant on the Stokes line, while the other solution is sub-dominant. Analogously, one can define “anti-Stokes lines” associated with each simple zero of Q^2 . On anti-Stokes lines, $Q(r)dr$ is purely real, which means that the two solutions are purely oscillatory. As we cross an anti-Stokes line, the dominance of the two functions $\Psi_{1,2}$ changes.

Stokes lines are vital for WKB analysis, because the solution changes character in the vicinity of these contours. That is, if the solution is appropriately represented by a certain linear combination of Ψ_1 and Ψ_2 in some region of the complex r -plane, the linear combination will change as the solution is extended across a Stokes line. The induced change is not complicated: The coefficient of the dominant solution remains unchanged, while the coefficient of the other solution picks up a contribution proportional to the coefficient of the dominant solution. This is known as the “Stokes phenomenon” [43]. The constant of proportionality is known as a “Stokes constant”. This change is necessary for the particular representation (2.12) to preserve the monodromy of the global solution. Terms that are exponentially small in one sector of the complex plane may be overlooked. However, in other sectors they can grow exponentially and dominate the solution. By incorporating the Stokes phenomenon, we have a formally exact procedure which leads to a proper account of all exponentially small terms.

In the particular case of an isolated simple zero of Q^2 the problem is straightforward. We choose the phase of the square-root of Q^2 such that

$$Q = R^{1/2} \sim \omega \quad \text{as } r \rightarrow \infty. \quad (2.17)$$

This means the boundary conditions which are the outgoing-wave solution at infinity is proportional to Ψ_1 while the ingoing-wave solution at the horizon is proportional to Ψ_2 . Suppose that the solution in the initial region of the complex plane is given by

$$\Psi = c\Psi_1^{(s)}. \quad (2.18)$$

Then, after crossing a Stokes line emanating from s (and on which Ψ_1 is dominant) the solution becomes

$$\Psi = c\Psi_1^{(s)} \pm ic\Psi_2^{(s)}. \quad (2.19)$$

The sign depends on whether one crosses the Stokes line in the positive (anti-clockwise) or negative (clockwise) direction. It is crucial to note that this simple result, i.e. that the Stokes constant is $\pm i$, only holds when the Stokes line emanates from the zero that is used as lower limit for the phase-integral. That is, when we want to use the above result to construct an approximate solution valid in various regions of the complex plane, we often change the reference point for the phase-integral. In this case, it is necessary to evaluate integrals of the type

$$\gamma_{ij} = \int_{s_i}^{s_j} Q(r)dr, \quad (2.20)$$

where s_i and s_j are two simple zeros of Q^2 .

Let us evaluate the above integrals. Near $r \simeq \pm iq$, we evaluate the phase-integrals

$$\begin{aligned} I &\equiv \int Qdr, \\ &\simeq \pm \int \frac{(r^2 + q^2)^2}{q^4} \left(\omega^2 + 9 \frac{q^{10}}{(r^2 + q^2)^6} \right)^{1/2} dr, \\ &\simeq \mp \int \frac{4(r \mp iq)^2}{q^2} \left(\omega^2 - \frac{9}{64} \frac{q^4}{(r \mp iq)^6} \right)^{1/2} dr. \end{aligned} \quad (2.21)$$

If we define,

$$y = \frac{8\omega(r \mp iq)^3}{3q^2}, \quad (2.22)$$

the zeros of Q map to -1 or 1 , and we can get

$$\begin{aligned} I &= \mp \frac{1}{2} \int (1 - \frac{1}{y^2})^{1/2} dy, \\ &= \pm \frac{\pi}{2}. \end{aligned} \quad (2.23)$$

Then, we obtain

$$\begin{aligned} \gamma &= -\gamma_{12} = -\gamma_{32} = \gamma_{43} = -\gamma_{54}, \\ &= -\gamma_{1'2'} = -\gamma_{3'2'} = \gamma_{4'3'} = -\gamma_{5'4'}, \\ &= \pi/2, \end{aligned} \quad (2.24)$$

where the lower indices are related to the zeros in Fig. 1.

Now we compute the QNMs utilizing “Stokes phenomenon”. For frequencies $|\text{Im } \omega| \gg |\text{Re } \omega|$, the pattern of Stokes and anti-Stokes lines is sketched in Fig. 1. Assuming that $\text{Re } \omega M > 0$ the outgoing wave boundary condition at spatial infinity can be analytically continued to the anti-Stokes line labeled a in the figure. The method to obtain these lines is discussed in more detail, for example, [44]. In order to obtain the WKB condition for highly damped QNMs, we analytically continue the solution along a closed path encircling the pole at the event horizon. This contour starts out at a , proceeds along anti-Stokes lines and account for the Stokes phenomenon associated with the zeros and eventually ends up at a . In other words, we choose the path

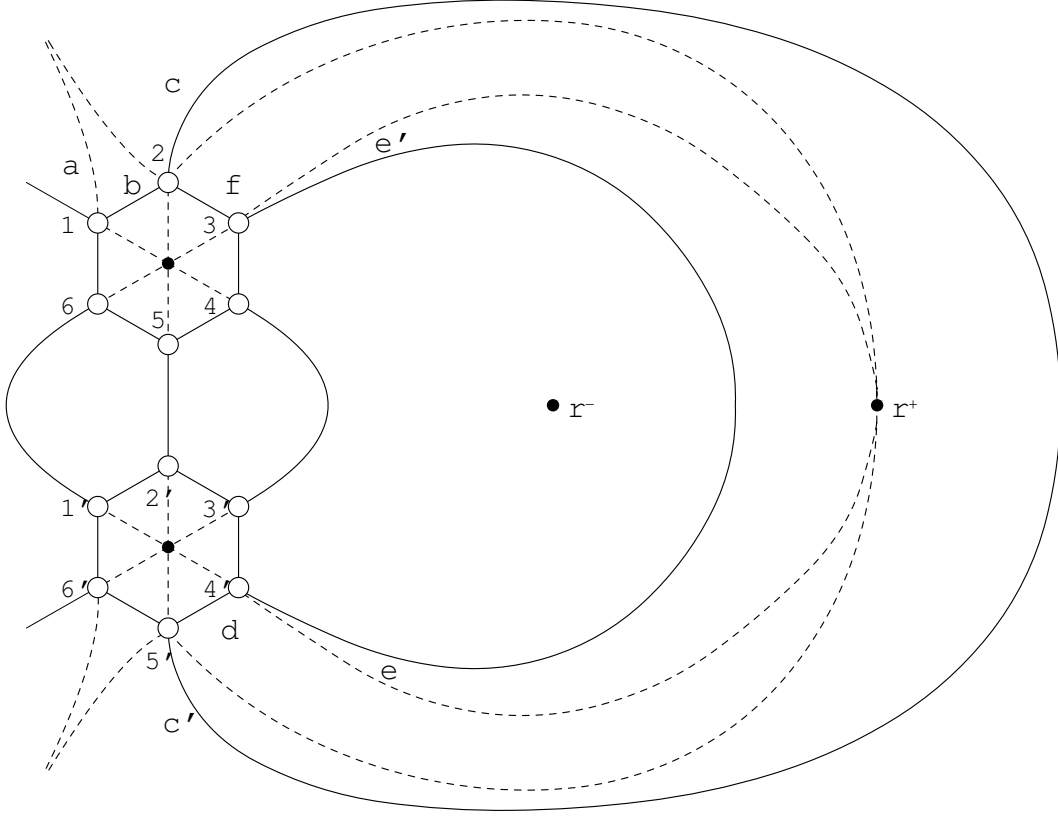


FIG. 1: A schematic illustration of the Stokes (dashed) and anti-Stokes (solid) lines for the regular black hole problem in the complex r -plane in the limit $\text{Im}(\omega) \rightarrow -\infty$. The open and filled circles represent zeros and poles of $Q^2(r)$ respectively.

$a \rightarrow b \rightarrow c \rightarrow c' \rightarrow d \rightarrow e \rightarrow e' \rightarrow f \rightarrow b \rightarrow a$ in Fig. 1. We can arrive at the following WKB condition for highly damped QNMs

$$e^{2i\Gamma_+} = 1 - (1 + e^{-2i\gamma})(1 + e^{2i\gamma})(1 + e^{-2i\Gamma_-}) \quad (2.25)$$

(We can perform the calculation quite analogous to the case in the Reissner-Nordström black hole explained in the Appendix of [15].), where Γ_+ and Γ_- denote the integral along a contour that encircles, in the negative direction, the pole at r_+ and r_- respectively. Substituting $\gamma = \pi/2$ into (2.25), we can get

$$e^{2i\Gamma_+} = 1. \quad (2.26)$$

Note that this expression is independent of Γ_- unlike the Reissner-Nordström case. This means that inner horizon does not contribute to the QNMs.

We evaluate the integral Γ_+ using the residue theorem

$$\begin{aligned} \Gamma_+ &= -2\pi i \text{Res} Q(r_+) , \\ &= -2\pi i \lim_{r \rightarrow r_+} \frac{r - r_+}{g(r)} \omega \sqrt{1 + \frac{g'(r)^2}{4\omega^2}} , \\ &= -2\pi i \lim_{r \rightarrow r_+} \frac{r - r_+}{g(r)} \omega , \\ &= -2\pi i \lim_{r \rightarrow r_+} \frac{1}{g'(r)} \omega , \\ &= -\frac{\omega}{2T_H} i , \end{aligned} \quad (2.27)$$

where we use $|\omega| \rightarrow \infty$ and (2.6). Then WKB condition can be written as

$$e^{\omega/T_H} = 1 . \quad (2.28)$$

We can get immediately

$$\omega = 0 - i \cdot 2n\pi T_H \quad n \rightarrow \infty. \quad (2.29)$$

Note that this result apply to scalar, electromagnetic, and odd parity gravitational perturbations.

III. CONCLUSIONS AND DISCUSSION

Our results show that the real part of the frequencies is zero in the highly damped limit. What does this mean?

Dreyer identified dA of (1.2) with A_{\min} in (1.3) obtained from loop gravity. However, the original Bohr's correspondence principle is as follows: "transition frequencies at large quantum numbers should equal classical oscillation frequencies". In other words, the distance between two neighboring energy levels ΔE with large quantum numbers (between levels with n and $n+1$ ($n \gg 1$)) is related to the classical frequency ω in the system by the relation $\Delta E = \hbar\omega$. Since the area spectrum of loop gravity is given by (1.3), spacing of the neighboring area spectrum ΔA , in general, approaches zero asymptotically in the classical limit (when A is sufficiently large). From the first law of black hole thermodynamics (1.2), it seems that the vanishing real part of ω supports continuous area spectrum applying to the original correspondence principle. In another context, Alexandrov and Vassilevich discuss that continuous area spectrum follows from the Lorentz covariant loop quantum gravity [45].

Let us interpret the QNMs for Schwarzschild black hole or other single-horizon black holes in this context. It has been discussed that one should take into account only the states with the minimal spin at the horizon counting of black hole states [32]. In this case, spacing of neighboring area spectrum ΔA does not approach zero and coincide with A_{\min} . If this idea applies to single-horizon black holes only, we can interpret the regular black hole and single-horizon black holes simultaneously. However, since it is still difficult to understand the results of the Reissner-Nordström and Kerr black holes, it is too early to conclude. Moreover, our analysis is only one example of QNMs of regular solutions. The absence of $r=0$ singularity may cause the existence of zero real part of ω [46]. Therefore, we need to investigate other regular solutions. Of course, it is also important to reconsider

the number counting of horizon states [21, 22, 47].

The subject is still debatable from a QNMs viewpoint. QNMs boundary conditions are stated in terms of behavior of perturbations at the horizon and infinity. It is somewhat strange that black hole quantization should care about infinity. Birmingham and Carlip show that these boundary conditions for the BTZ black hole can be recast in terms of monodromy conditions at the inner and outer horizons and define a set of "non-QNMs" for the higher-dimensional black holes involving only these monodromies [48]. The correspondence principle leads to the correct quantization of the near-horizon Virasoro generators. Boundary conditions for the inner horizon and outer horizon might be the key to solve the problem. This gives a suggestion that single-horizon black holes have consistency, while dual horizon black holes, such as Reissner-Nordström and Kerr black holes, have no consistency. We need further discussion from various points of view.

Acknowledgments

We would like to thank Shijun Yoshida and Ryo Miyazaki for useful comments and help. This work was partially supported by The 21st Century COE Program (Holistic Research and Education Center for Physics Self-Organization Systems) at Waseda University. This work was supported in part by Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Science, Culture and Technology of Japan, 2003, No. 154568 (T.T.). This work was also supported in part by a Grant-in-Aid for the 21st Century COE "Center for Diversity and Universality in Physics".

-
- [1] K.D. Kokkotas and B.G. Schmidt, Living Rev. Rel. **2**, 2 (1999).
 - [2] J. Bekenstein, Lett. Nuovo. Cim. **11**, 467 (1974); in Cosmology and Gravitation, edited by M. Novello (Atlasciences, France, 2000), pp. 1-85, gr-qc/9808028 (1998).
 - [3] S. Hod, Phys. Rev. Lett. **81**, 4293 (1998).
 - [4] L. Motl, Adv. Theor. Math. Phys. **6**, 1135 (2003); L. Motl and A. Neitzke, *ibid.*, **7**, 307 (2003).
 - [5] C. Rovelli and L. Smolin, Nucl. Phys. B **442**, 593 (1995); Erratum, *ibid.*, **456**, 753 (1995).
 - [6] G. Immirzi, Nucl. Phys. Proc. Suppl. B **57**, 65 (1997).
 - [7] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998).
 - [8] O. Dreyer, Phys. Rev. Lett. **90**, 081301 (2003).
 - [9] A. Alekseev, A. P. Polychronakos, and M. Smedback, Phys. Lett. B **574**, 296 (2003); A. P. Polychronakos, Phys. Rev. D **69**, 044010 (2004).
 - [10] A. Corichi, Phys. Rev. D **67**, 87502 (2003).
 - [11] I.B. Khriplovich, Int. J. Mod. Phys. D **14**, 181 (2005).
 - [12] R. K. Kaul and S. K. Rama, Phys. Rev. D **68**, 24001 (2003).
 - [13] A. Corichi, Rev. Mex. Fis. **50**, 549 (2005).
 - [14] Y. Ling and L. Smolin, Phys. Rev. D **61**, 04408 (2000); Y. Ling and H. Zhang, Phys. Rev. D **68**, 101501 (2003).
 - [15] N. Andersson and C. J. Howls, Class. Quantum Grav. **21**, 1623 (2004).
 - [16] E. Berti and K. D. Kokkotas, Phys. Rev. D **68**, 044027 (2003).
 - [17] R. A. Konoplya, Phys. Rev. D **66**, 044009 (2002); V. Cardoso, R. A. Konoplya, and J. P. S. Lemos, Phys. Rev. D **68**, 044024 (2003); V. Cardoso and J. P. S. Lemos, Phys. Rev. D **67**, 084020 (2003); A. Maassen van den Brink, Phys. Rev. D **68**, 047501 (2003); R. A. Konoplya and A. Zhidenko, JHEP **0406**, 037 (2004); V. Cardoso, J. Natário, and R. Schiappa, J. Math. Phys. **45**, 4698 (2004).
 - [18] S. Yoshida and T. Futamase, Phys. Rev. D **69**, 064025 (2004).
 - [19] S. Hod, Phys. Rev. D **67**, 081501 (2003); E. Berti, V. Cardoso, K. D. Kokkotas, and H. Onozawa, Phys. Rev. D **68**, 124018 (2003); E. Berti, V. Cardoso, and S. Yoshida, Phys. Rev. D **69**, 124018 (2004); M.R. Setare and E. C. Vagenas, hep-th/0401187 (2004).
 - [20] R. Schiappa, hep-th/0411267 (2004).

- [21] M. Domagala and J. Lewandowski, *Class. Quant. Grav.* **21**, 5233 (2004).
- [22] K.A. Missner, *Class. Quant. Grav.* **21**, 5245 (2004).
- [23] G. Kunstatter, *Phys. Rev. Lett.* **90**, 161301 (2003).
- [24] D. Birmingham, *Phys. Lett. B* **569**, 199 (2003).
- [25] V. Cardoso, J. P. S. Lemos, and S. Yoshida, *Phys. Rev. D* **69**, 044004 (2004).
- [26] G. W. Gibbons and K. Maeda, *Nucl. Phys. B* **298**, 741 (1988); D. Garfinkle, G. T. Horowitz and A. Strominger, *Phys. Rev. D* **43**, 3140 (1991).
- [27] T. Tamaki and H. Nomura, *Phys. Rev. D* **70**, 044041 (2004).
- [28] A. J. M. Medved, D. Martin, and M. Visser, *Class. Quantum Grav.* **21**, 1393 (2004); *ibid.*, 2393 (2004).
- [29] T. Padmanabhan, *Class. Quantum Grav.* **21**, L1 (2004); T. R. Choudhury and T. Padmanabhan, *Phys. Rev. D* **69**, 064033 (2004).
- [30] J. Kettner, G. Kunstatter, and A.J.M. Medved, *Class. Quant. Grav.* **21**, 5317 (2004).
- [31] S. Das and S. Shankaranarayanan, *Class. Quant. Grav.* **22** (2005) L7.
- [32] S. Alexandrov, gr-qc/0408033 (2004); O. Dreyer, F. Markopoulou and L. Smolin hep-th/0409056 (2004).
- [33] G. T. Horowitz and V. E. Hubeny, *Phys. Rev. D* **62**, 024027 (2000).
- [34] D. Birmingham, I. Sachs, and S. N. Solodukhin, *Phys. Rev. Lett.* **88**, 151301 (2002).
- [35] J. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
- [36] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [37] D. Birmingham, S. Carlip and Y. Chen, *Class. Quant. Grav.* **20**, L239 (2003).
- [38] D. Birmingham, *Phys. Lett. B* **569**, 199 (2003).
- [39] R.A. Konoplya, *Phys. Rev. D* **71**, 024038 (2005).
- [40] E. Ayón-Beato and A. García, *Phys. Rev. Lett.* **80**, 5056 (1998); *Phys. Lett. B* **464**, 25 (1999); *Gen. Rel. Grav.* **31**, 629 (1999); *Gen. Rel. Gravit.* **37**, 635 (2005).
- [41] N. Breton, *Phys. Rev. D* **67**, 124004 (2003).
- [42] R.E. Langer, *Phys. Rev.* **51**, 669 (1937).
- [43] M. V. Berry, *Proc. Roy. Soc. A* **422**, 7 (1989).
- [44] N. Andersson, M.E. Araújo and B.F. Schutz, *Class. Quantum Grav.* **10**, 735 (1993).
- [45] S. Alexandrov and D. Vassilevich, *Phys. Rev. D* **64**, 044023 (2001).
- [46] A. Maassen van den Brink, *Journal of Math. Phys.* **45**, 327 (2004); *ibid.*, *Phys. Rev. D* **62**, 064009 (2000); P.T. Leung, A. Maassen van den Brink, K. W. Mak and K. Young, gr-qc/0307024 (2003).
- [47] I.B. Khriplovich, gr-qc/0409031; gr-qc/0411109; A. Dasgupta, JCAP08, 004(2003).
- [48] D. Birmingham and S. Carlip, *Phys. Rev. Lett.* **92**, 111302 (2004).